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DISCOVERIES
IN
MATHEMATICS,
INTRODUCING
NEW PRINCIPLES, FORMULÆ, & DOUBLE EQUATIONS,
WHICH ABRIDGE
ALL THE OPERATIONS
OF
ALGEBRA & ARITHMETIC.
DESIGNED FOR THE USE OF
HIGH SCHOOLS, COLLEGES, & ACADEMIES.

THIS TREATISE IS AN ENLARGEMENT OF A PAPER, READ BEFORE
THE UNIVERSITY CONVOCATION OF THE STATE OF N. Y.
AT ALBANY N. Y. July 13th 1880.

HEYER M. NEXSEN, AUTHOR AND PUBLISHER.
OFFICE, 80 VESEY STREET, ROOM No. 14, N. Y.
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PRICE of Nexen's Short-Hand Arithmetic, (*a large work*) $ 2.50. Discount to trade, to Teachers, Schools and Colleges.

PREFACE.

By the request of Erastus C. Benedict, L L. D., *late Chancellor of the University*, the following treatise is printed in full, extracts from which were read by the present writer as a paper, titled " *RECENT DISCOVERIES IN ALGEBRA AND ARITHMETIC* " before the **UNIVERSITY CONVOCATION OF THE STATE OF NEW YORK**, July 13th 1880, at Albany N. Y.

Principles and formulæ are presented in this work, which will **ABBREVIATE ALL THE OPERATIONS OF ALGEBRA AND ARITHMETIC**, ***TO A VAST EXTENT***, and make it possible to learn or teach them both together, or separately if prefered ; and *in far less time* than it takes to learn either of them in the ordinary way ; for *whatever abbreviates Algebra, should also abbreviate Arithmetic*, because the same principles govern both, or should govern both. We will show how this can be done, by the Introduction.

INTRODUCTION.

How to Reduce Algebraic Operations.

It is by a **CONDENSATION** of algebraic terms, that this extraordinary power is gained, which is lost by algebraic expansion. See page 11.

For instance, the Newtonian formulæ, can be condensed to ***half as many terms,*** as the reader will note on pages 28 and 32 to 36.

The power which these new formulæ possess, is made evident by examples on pages 23, 26, 27 and 31.

And the law for the formation and development of the *Coefficients, Exponents, and Signs*, set forth on pages 33 to 36, is not only curious, but very important to the mathematician, to aid him in finding the Powers and Roots of numbers with **greater facility** than by any other process.

By the use of **DOUBLE EQUATIONS**, we **CANCEL TWO-THIRDS OF the usual ALGEBRAIC OPERATIONS**, and this is explained on page 12, and illustrated on pages 19 and 20, by algebraic formulæ and several examples in arithmetic.

Algebraic Formulæ which Reduce Arithmetical Operations.

So that by *deductions from algebra,* like those on pages 11 and 24, and by *reductions of algebra* like those on pages 12, 30 and 32, we find formulæ which facilitate every branch of Arithmetic.

Logarithms were invented to facilitate Multiplication, Division, Involution and Evolution, and not to facilitate Addition or Subtraction.

But we propose to present principles which will surpass logarithms in their functions, and do besides, what can not be done by logarithms.

A Principle To Facilitate Many Operations.

Addition, Multiplication and Involution can be performed *by the Addition or Multiplication of exactly the same terms*, according as the case requires. See the pages 19, 20, and 22.

By ***simplifying Addition, we necessarily simplify Multiplication, Involution, etc.***

By the operation of one principle, we facilitate Addition, Multiplication, the Powers and Roots of Numbers, Compound Interest and Discount, Insurance, Annuities, Mensuration, and whatever else depends upon Involution and Evolution, etc. ; and this we do by *finding a similarity of terms* which exist among these different branches : and thus *reduce them all to a common basis.* See pages 19 and 22.

By another principle, we facilitate Subtraction, Division, certain kinds of Multiplication, and various branches of Arithmetic.

Integration and Differentiation of Factors.

We also find that the **SUM** and **DIFFERENCE** of the **FACTORS THEMSELVES**, ***CONTAIN THE TERMS*** already alluded to, which are simpler than the given factors or the logarithms devised to represent the given factors. See pages 19 to 21, 24 and 27.

Multiplication based upon the Addition of Factors, instead of the Addition of their Logarithms.

So that Multiplication is performed by the Addition of the factors themselves, rather than by the Addition of logarithms ; because, *as Any Product or Power contains its factors, they must therefore contain the sum of their factors.*

Powers and Roots dependent on the Same Principle.

In this manner, we abridge the labor of finding the Powers and Roots of Numbers to a vast extent ; because the sum of any number of equal factors which a Power contains, is expressed by simply multiplying the number itself which is to be raised and not its logarithm, by the Index of Power. See page 27. And this quantity will exactly contain the sum of whatever factors will more readily produce the required Power.

We therefore *divide* the sum of the factors of the divisors and quotient, which a Power contains, (instead of dividing the logarithm of that power), by the Index of the required Root, and thus ascertain the Root required. See pages 24 to 27, which also contain a curious algebraic illustration of this method.

Advantages Of This System.

Among the advantages of this new system, we would invite particular attention, first—to the fact, that *in finding the Roots of numbers* ***MANY ROOT FIGURES ARE ASCERTAINED ALL AT ONCE,*** by short division, double as many in the next, and so on, ***REACHING FAR BEYOND THE PRESENT LIMITS OF LOGARITHMS,*** ecially where the root figures are interminable, (see page 23), and s is also a decided advantage over the tedious way of finding only figure at a time as is customary.

gain, logarithms do not aid us in finding more than six or ht figures of an answer, when more would be desired, as in ding high powers, or in performing great multiplications, etc., ereas this new system produces many more.

ALL LIABILITY TO ERROR AVOIDED.

nother advantage in this system, is, that *as it shortens every culation exceedingly, IT AVOIDS ALL LIABILITY TO ERROR,* d is therefore far more reliable than any other: and this we im, to be a desideratum of incalculable value, so much sought . by the greatest mathematcians, such as Lord Napier.

new process for finding the cubes of numbers, is presented on ge 31.

The new rule for squaring, on page 22, will be found to apply to mbers which cannot be performed by the usual formula.

Geometrical demonstration of this method of squaring, on page will be found to be very interesting.

ADDITION ALSO, GREATLY FACILITATED.

esides, the principle which reduces so many different operations a common basis, enables us also, to discover and classify all mbinations of figures and numbers, and will facilitate addition in most extraordinary manner.

We invite the reader to consider the tables of combination prented on pages 14 and 15, as they will enable him to perform dition with a rapidity, far beyond ordinary conception.

For, as millions of numbers can be resolved into ten figures, so illions of combinations of combinations, can be resolved into those the tables of combinations just refered to.

OTHER PRINCIPLES.

Having demonstrated largely in this work, the extraordinary power ich a certain principle possesses, to facilitate so many different erations of Algebra and Arithmetic, by reducing them to a mmon basis, we could also both algebraically and arithmetically ow, that by simplifying Subtraction, we can simplify Division, e Subtraction of Compound Numbers, certain kinds of ultiplications, etc., see **NEXSEN'S SHORT-HAND ARITHMETIC**, which eats upon Algebra and Arithmetic, and shows the value of abbreating other Algebraic formulæ, and contains many other methods calculation.

We hope also, that the learned and wise will see the many abvanges in teaching a system which grasps so many different subjects ider each principle; which will facilitate education so much, and hich *EXERCISE AND DEVELOP THE REASONING POWERS* the mind to a very *fine degree*: and trust that our colleges will lopt this work as the *best system of algebraic training* yet produced.

We present on page 9 an example to show the superiority the new method over the old.

This treatise is not intended to supercede any Algebra now use. but to show the benefit of a more highly finished cou and the practicability of making Algebra far more useful.

The author would express his thanks for the kindness sho by G. B. Docharty, Esq. formerly Professor of Mathematics the College of the City of New York, for the introduction of t subject to the attention of the Chancellor of the University of t State of N. Y. ; and thereby bringing it prominently before t University Convocation.

The author would also beg to present his thanks to Pr M. A. NEWELL, State Superintendent of the Schools of Maryla and President of the State Normal School for Teachers of s State, for his personal introduction of the author, and of discoveries before their publication, in the Convention of t Teachers Association of the State of Maryland, which assembl at Cumberland Md.

The late Professor JOSEPH HENRY, of the Smithsonian Instit tion at Washington, manifested a deep interest in this subject, a desired to bring it before the Washington Philosophical Socie by inviting the author to meet this honorable body, to set fo the merits of this work ; but the unexpected demise of one the members, prevented its presentation to said body.

The author would be pleased to render his assistance to a College, High School or Academy, desiring to introduce this wo

NEW METHOD. What is the eleventh root of 11 ? *See page* 28.

$(11)^6$=1771561)11.
$(14)^3$= 2744)6.209 its 5th root is 1.4
$(146)^2$= 2.1316= 2.15, its 2nd root is 1.46
146 × 2= 292
14 × 3= 42
11 × 6= 66
11)1372
1.24 THE ROOT.

$(12)^6$=2985984)11.
$(13)^3$= 2197) 3.6838777 its 5th R.=1.3
$(129)^2$=1.6641= 1.676776 its 2nd R.=1.29
129 × 2= 258
13 × 3= 39
12 × 6= 72
11)1368
1.2436– ROOT CONTINUED.

OLD METHOD. What is the eleventh root of 11 ? *See Perkins' Arith.*

1st Col.	2d Col.	3d Col,	4th Col.	5th Col.	6th Col.	7th Col.
1	1	1	1	1	1	1
2	3	4	5	6	7	8
3	6	10	15	21	28	36
4	10	20	35	56	84	120
5	15	35	70	126	210	330
6	21	56	126	252	462	4438023168
7	28	84	210	462	569011584	5822306304
8	36	120	330	53505792	692141568	7488297728
9	45	165	3652896	61564992	832995712	9474862720
10	55	176448	4029600	70427072	993282496	11824496640
112	5724	188352	4431040	80143392	1174816960	123961454696751104
114	5952	200720	4858160	90767232	1379524608	129650860737626112
116	6184	213560	5311920	102353824	1426622074167776	
118	6420	226880	5793296	114960384	1474651510218752	
120	6660	240688	6303280	117749665546944		
122	6904	254992	6842880	120573590007744		
124	7152	269800	69682036736			
126	7404	285120	70748115200			
128	7660	288309184				
130	7920	291519616				
1324	797296					
1328	802608					

8th Col.	9th Col.	10th Col.
1	1	1
9	10	11
45	55	321504185344
165	105752092672	681091006464
25376046336	179793410560	8067509802924592070656
37020658944	283787919360	945|2868057140984283136
51997254400	314149934571148017664	
70946979840	346589563554096053120	
7590503802787004416		
8109907245737506864		

11(1.2436. nearly, which is a trifle too great.
1
10
643008370688
856991629312
82270039211696368282624
8429128719601631717376
2836
593
567
26

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Abbreviations of Algebraic Formulæ By CONDENSATION OF TERMS.

Let $M=(a+x)$, then $a+x=M$. and $a=M-x$. Remember this value of a.

Let $N=(b+y)$, then equal successively, $a+y$, $2a+y$, $3a+y$, $\frac{1}{2}a+y$., etc.

Multiplying the different values of N, by M, we find a ***series of briefer formulæ, fitter for mathematical and practical calculations*** **when the conditions of their application are made known. Then, N times M, will produce the following**

CONTRACTED FORMULÆ.

$(b+y)(a+x)=a(b+y)+bx+xy=aN+bx+xy$, A GENERAL FORMULÆ. 1.

$(a+y)(a+x)=a(a+y)+ax+xy=a(N+x)+xy$ $(M-x)(N+x)+xy$, if $b=a$. 2.

$(2a+y)(a+x)=a(2a+y)+2ax+xy=a(N+2x)+xy=$ $M\times N$, when $b=2a$. 3.

$(3a+y)(a+x)=a(3a+y)+3ax+xy=a(N+3x)+xy=$ $M\times N$, when $b=3a$. 4.

$(4a+y)(a+x)=a(4a+y)+4ax+xy=a(N+4x)+xy=$ $M\times N$, when $b=4a$. 5.

$(\frac{1}{2}a+y)(a+x)=a(\frac{1}{2}a+y)+\frac{1}{2}ax+xy=a(N+\frac{1}{2}x)+xy=$ $M\times N$, when $b=\frac{1}{2}a$. 6.

$(\frac{1}{3}a+y)(a+x)=a(\frac{1}{3}a+y)+\frac{1}{3}ax+xy=a(N+\frac{1}{3}x)+xy=$ $M\times N$, when $b=\frac{1}{3}a$. 7.

$(\frac{3}{4}a+y)(a+x)=$ $a(N+\frac{3}{4}x)+xy$, when $b=\frac{3}{4}a$. 8.

$(\frac{2}{3}a+y)(a+x)=$ $a(N+\frac{2}{3}x)+xy$, when $b=\frac{2}{3}a$. 9.

As $a=M-x$, let $T=N+x$; then formula number (2) becomes

$$(a+y)(a+x)=aT+xy.$$

by which we learn, that,

When any two factors, (M and N) *are* ***EQUALLY DIMINISHED AND INCREASED*** by any quantity or number (x); the results $M-x$ and $N+x$, will be *reversely equidistant factors* expressed by the quantities a and T. **Hence**

The Product of any Two Numbers, *is equal to the product of* ***any*** *two other numbers which are* ***reversely*** *equidistant from the given numbers plus or minus the value of xy,* which we will find next.

$$\begin{array}{lll} \text{As } N=a+y. \text{ then} & N-a=y= & \textit{the first difference.} \\ \text{As } M=a+x. \text{ then} & M-a=x= & \textit{the second difference} \\ \hline \text{Hence} & (M-a)(N-a)=xy. & \textit{Prod. of Diffs.} \end{array}$$

So that, x and y, express the differences between one of those equidistant numbers (a), when it is compared to both factors; for N and M are both diminished by (a), and leave x and y.

Hence, ***The Product of Any Two Numbers,*** *is equal to the product of any two other numbers, which are* ***reversely*** *equidistant from them, (produced by an* ***EQUAL DIMINUTION AND INCREASE OF THE GIVEN FACTORS***), *plus or minus the relative* ***differences which both given factors bear to*** (a), *or to* ***either one of the equidistant numbers.***

As $(a+y)(a+x)=aT+xy$. we could show by similar reasoning, that
$(a+y)(a-x)=aT-xy$. and that
$(a-y)(a-x)=aT+xy$. etc., which will hereafter be illustrated with examples: but will continue with these new theories first, in order to show a better system of contracting algebraic formulæ. Thus

Express a multiplicand, by two equivalents, as $N = T-x = a+y$, and make each of them contain a different term of the equivalent of the multiplier $a+x$, which we will repeat; and arrange the equations, so that like terms shall stand under like terms, and unlike terms under unlike terms. Then

Cancel the like terms, and multiply the unlike, and the result will be the product of any two equivalents.

REASON. For $(a+x)$ times $T-x$, we first multiply $T-x$, by a, the first term of $a+x$, and obtain $aT-ax$: we now require only x times $T-x$, to complete the multiplication; but $T-x$ is equal to $a+y$, and x times the former, is x times the latter, because they are equals, either of which we can take, for if equals be multiplied by equals, their products will be equal; prefering then x times $a+y$ which is $ax+xy$, and adding it to $aT-ax$, we find ax disappears, thus, $aT-ax+ax+xy = aT+xy$, so that only the unlike terms have been multiplied.

This is not only curious but very many unnecessary algebraic multiplications are avoided, and the result is obtained direct, without them.

For by double equation, we find that in the multiplication of $T-x$ by $a+x$, which is $aT-ax+Tx-x^2$, we only retain the first term aT, and discard the three following terms of $-ax+Tx-x^2$, because they are useless and the multiplications which produced them are also useless. Again, we find that in the mulitplication of $a+y$ by $a+x$, which is $a^2+ay+ax+xy$, we only retain the last term xy, and discard the three preceding terms of $a^2+ay+ax$, because they are useless, and the multiplications which produced them are also useless.

Hence the products of $aT-ax+Tx-x^2$ and $a^2+ay+ax+xy$, would only yield $aT+xy$ of any value, which we can get without performing so many multiplications to be afterwards discarded.

So we see, that by double equations we dispense with three algebraic multiplications out of every four required in multiplying any two equivalents, and for this reason, we claim that by double equations, we not only avoid many superfluous algebraic multiplications, but obtain the result immediately, by cancelling the like terms and multiplying only the unlike which remain.

We notice that these double equations are produced firstly, by increasing and diminishing the given numbers by the same quantity (x), to produce the first set of equivalents. Thus $N+x$ producing T. and $M-x$ producing a. Secondly, by comparing both factors to the first term of the multiplier (a), we have $N=a+y$, and $M=a+x$, and find we must add y and x to (a), to form the second set of couplets, and notice that the multiplier is repeated, while the multiplicand is expressed by two different equivalents.

Let $a+y=N$, then $a+y+x = N+x = T$, then $N = T-x = a+y$ besides.
Let $a+x=M$, then $a = M-x = a$, then $M = a+x = a+x$ again.
$a^2+ay+ax+xy = a(a+y+x)+xy = (M-x)(N+x)+xy = MN = aT + xy$

Hence $(a+y)(a+x) = a(N+x)+xy = a \times T. + xy = NM.$ is formula No. 2.

Therefore $N \times M = (M-x)(N+x)+xy = a(N+x)+xy = aT+xy$; as formula No. 2.

We now wish to prove that the terms *a* and T, which we have already multiplied, are exactly those which satisfy addition, that is, that the terms which may be required for the addition of two quantities, are identically the same terms which are required for the multiplication of those quantities, and we do this, in order to show also, that by *the principle of* **AN EQUAL DIMINUTION AND INCREASE** of two numbers, we can curtail not only many algebraic multiplications, but also simplify to a vast extent, *by the very same principle*, Addition, Multiplication, and the Powers and Roots of numbers ; and we can also prove that by this principle, that Arithmetic and Algebra are not only both abbreviated to a remarkable degree, but that many **wasteful calculations can be avoided, eliminating therefore all liability to error, and obtaining direct results, with greater assurance of their accuracy**, and that this is the principle, which reduces Addition, Multiplication, and the Powers and Roots of numbers, all to the same **basis, and will enable us to calculate without the use of logarithms.**

APPLICATION OF THIS PRINCIPLE TO ADDITION.

If we add $N+x=T$, to $M-x=a$, the value of (x) disappears by cancelation, and we discover that $N+M=a+T$. but *a* and T are numbers equidistant from N and M, by adding x to N, and subtracting x from M. Therefore

***The Sum of Any Two Numbers**, is equal to the sum of any two other numbers which are reversely equidistant from the given numbers, by an EQUAL QUANTITY OF DIMINUTION AND INCREASE, that is, by taking from one number to increase the other ;* or others, when there are several numbers to add.

For if we *diminish one or more numbers, so as to increase others, MAKING THE AMOUNT OF ALL THE DIMINUTIONS, EQUAL TO THE AMOUNT OF ALL THE ADVANCES,* we discover easier numbers to add, even intuitively, producing tens, twenties, thirties, &c. or equalized numbers. Thus

$$
\begin{array}{r}
9+1=10\\
7+3=\underline{10}\\
4-4=0\\
\hline
20+0=20
\end{array}
\qquad
\begin{array}{r}
9+1=10\\
9+1=10\\
7+\underline{3}=10\\
8-5=3\\
\hline
33+0=33
\end{array}
\qquad
\begin{array}{r}
9-1=8\\
9-1=8\\
8\quad=8\\
7+1=8\\
7+1=8\\
\hline
40\quad=40
\end{array}
\qquad
\begin{array}{r}
97+3=100\\
89+\underline{11}=100\\
49-14=35\\
\hline
235+0=235
\end{array}
$$

By this principle, we discover all the combinations of numbers which addition involves, and have arranged them into series in tabular form, which are easy to learn, and will therefore simplify any of the most difficult additions that ever occur.

Notice the following Tables of Combinations, and see pages 35, 36, and 45, to 47 of 'NEXSEN'S SHORT-HAND ARITHMETIC' which treats this subject more fully, and will be found to be not only curious, but very useful and interesting.

By A Series Of Combinations, Amounts of which, Form A Scale Of Decimals.

1	2	3	4	5	6	7	8
.	.	.	.	.	.	.	9
.	.	.	.	.	.	9	9
.	.	.	.	.	9	9	9
.	.	.	.	9	9	9	9
.	.	.	9	9	9	9	9
.	.	9	9	9	9	9	9
.	9	9	9	9	9	9	9
9	9	9	9	9	9	9	9
1	2	3	4	5	6	7	8
10	*20*	*30*	*40*	*50*	*60*	*70*	*80*

Take the units of the smallest number, to increase the other numbers to decimals. As the figures in the margin, with the signs + and —, illustrate in a few cases in these Tables.

The amounts in this table will always be as many decimals, as one less than the number of numbers, which compose any combination.

9	10	11	12	13	14	15	
.	.	.	.	.	.	9	+ *1*=10
.	.	.	.	.	9	9	+ *1*=10
.	.	.	.	9	9	9	+ *1*=10
.	.	.	9	9	9	9	+ *1*=10
.	.	9	9	9	9	9	+ *1*=10
.	9	9	9	9	9	9	+ *1*=10
8	8	8	8	8	8	8	+ *2*=10
2	3	4	5	6	7	8	— *8*= 0
10	*20*	*30*	*40*	*50*	*60*	*70*	== 70

24	25	26	27	28
.	.	.	.	9
.	.	.	9	9
.	.	9	9	9
.	9	9	9	9
8	8	8	8	8
8	8	8	8	8
4	5	6	7	8
20	*30*	*40*	*50*	*60*

16	17	18	19	20
.	.	.	.	9
.	.	.	9	9
.	.	9	9	9
.	9	9	9	9
7	7	7	7	7
3	4	5	6	7
10	*20*	*30*	*40*	*50*

33	34	35	36
.	9	6	8
6	7	8	8
7	7	8	7
7	7	8	7
20	*30*	*30*	*30*

21	22	23
		9
	9	9
6	6	6
4	5	6
10	*20*	*30*

32
8
6
6
20

29	30	31	
.	.	9	+ *1*=10
.	9	9	+ *1*=10
8	8	8	+ *2*=10
7	7	7	+ *3*=10
5	6	7	— *7*= 0
20	*30*	*40*	— 40

As the complement of any number, is whatever that number requires to make it a decimal, thus the complement of 8 is 2, because it takes a 2 with an 8, to make a ten.

Then the complement of several numbers, is the sum of the complements of those numbers, showing how much they all require together, to make them all become decimals.

We therefore call such numbers as 10, 20, 30, and 400, decimals ; because they evidently contain a certain number of decimals, which the significant figures indicate.

Hence. ***When any number expresses the sum of the complements of several others, combining it with them,*** **is the same as taking from it, whatever they require to make them become decimals ; for the amount of the surplus additions made, is equal to the subtraction of a single figure, which that subtraction cancels, and the principle of equal diminution and increase, becomes evident.. Hence**

RULE FOR ADDITION.

Diminish one number, to increase several others to decimals ; add these results ; repeat this operation as often as is necessary ; then add the amounts thus obtained for the sum total. See examples page *13*, and many others in the author's work on *Short-Hand Arithmetic.*

Notice 1. The first series can be remembered, because an equal number of nines and units, will amount to the same number of tens. Thus, 9+9+9+3=30.

Notice 2. For every figure nine, in this Table of Combinations, there is a unit added to the complement in the first group which begins each series, two units added for two nines, three for three nines ; and thus, ***as many tens are added*** to the simple groups of 9 & 1, 8 & 2, 7 & 3, 6 & 4, of 8+8+4, &c., ***as the additional number of nines and units indicate. Thus,***

	2+8 =10	2+8 =10	4+8+8 =20
Adding	1 +9 =10	2 +9+9=20	1 +9=10
Makes	3+8+9 =20	4+8+9+9=30	5+8+8+9=30
Adding	1 +9=10		
Makes	4+8+9+9=30		

These Combinations can also be learned, by the simple process of doubling, tribling, etc. the combinations which amount to 10, such as 9+1, & 8+2, viz.

9+9+2=20, equals 2 nines and 2 ones, and is therefor twice (9+1).
8+8+4=20, equals 2 eights and 2 twos, and is therefor twice (8+2).
7+7+6=20, equals 2 sevens and 2 threes, and is therefor twice (7+3).
6+6+8=20, equals 2 sixes and 2 fours, and is therefor twice (6+4).

9+9+9+3=30, equals 3 nines and 3 ones, and is therefor 3 times (9+1).
8+8+8+6=30, equals 3 eights and 3 twos, and is therefor 3 times (8+2).
7+7+7+9=30, equals 3 sevens and 3 threes, and is therefor 3 times (7+3).

2 and 2 nines make 20, 3 and 3 nines make 30, etc., See Notice 1.

Doubling 2+9+9=20, we get 4+9+9+9+9=40. As Combination No *4*.
Doubling 3+8+9=20, we get 6+8+8+9+9=40. As Combination No *26*.
Doubling 4+7+9=20, we get 8+7+7+9+9=40. As Combination No *31*.

Doubling 7+8=15, we get 7+7+8+8=30, as combination No *36*.
Doubling 6+9=15, we get 6+6+9+9=30, as combination No *23*.

6+7+8+9=30 is twice 15, as (6+9)+(7+8). Remembered by the natural order of 6, 7, 8, and 9.

For others, see page 47 of NEXSEN'S SHORT-HAND ARITHMETIC.

RESOLVING COMBINATIONS INTO THEIR COMPONENT TENS.

As 3=2+1, then 9+8+3=9+8+(2+1)=(9+1)+(8+2)=20

As 5=3+2, then 9+8+8+5=9+8+8+(3+2)=(9+8+3)+(8+2)=20+10=30.

As 5=4+1, then 9+8+8+5=9+8+8+(4+1)=(9+1)+(8+8+4)=10+20=30.

9+5+6=(9+[1)+(4]+6)=20. Again 9+6+5=9+(6+[3)+2]=20. Etc.

So that great combinations, difficult to add in the ordinary way, are only a combination of two or more simple ones, by a condensation of a single term from each. Let the pupil learn to construct some of these combinations, and he will easily remember them.

To find how many nines, any of these combinations should contain.

When there is a single 8, associated with any figure greater than 2; the complement of 8: *Take the complement of 8,* (which is 2) *from the figure which exceeds it, and the remainder will designate how many nines combined with that figure, and the 8,* will make a certain number of tens; as in the second series at the left-hand of the table.

Thus 4+8, being 2 more than 2+8 associate *two* nines, we get 4+8+9+9=30.

When there are two eights, associated with any figure greater than twice the complement of 8, (which is 4,). *Take 4, (the sum of the complements of two eights,) from that figure, and the remainder will designate the number of nines, which, combined with that figure and the two eights,* will make a certain number of tens. Thus 5+8+8, being 1 more than 4+8+8, associate only *one* nine, we get 5+8+8+9=30.

TRANSPOSITION OF FIGURES.

Any Combination of figures difficult to add, can be found in this table, by a simple transposition of some of them, thus 8+3+9, or 3+8+9, amount to the same, as 9+8+3=20. 4+9+9+8=9+9+8+4=30.

Notice. Some of these combinations can also be remembered by the next table. Thus 9+9++7+7+8=9+9+8+7+7=40, because, if we take a unit, from each nine, to add to each seven, we make them all eights; and five eights equal 40.

Notice. Some of these combinations, can be remembered by a multiplication. Thus, 9+9+9+9+8+6 is equal to 4 times 9, with 8+6; equal to 36+8+6, and equal to 30+(6+8+6)=30+20=50.

These series form a methodical arrangement, and classification of an endless variety of combinations, which (by transposition) can easily be recognized and remembered, by their systematic order.

For, if millions of numbers can be formed by only ten figures, how many millions of combinations of combinations, can be formed by 55 combinations of figures? It is wonderful.

By this Scale of Decimal Combinations, *vast power* is gained over numbers; and addition is performed by strides or leaps, as it were.

A Scale of Decimals, is like a scale of weights, by which to weigh any bulk or quantity.

So numbers of all magnitudes, can be measured by a Scale of Decimals, like 10, 20, 30, 40, etc., 100, 200, 300, 400, etc., See the author's work on SHORT-HAND ARITHMETIC, for a full treatment of addition.

FURTHER USES OF THESE COMBINATIONS.

Because...... 9+9+2=20,
then.......... 9+9+3=21, as 9+9+3, is *1* more than 9+9+2,
9+9+4=22, as 9+9+4, is *2* more than 9+9+2.
9+9+5=23, as 9+9+5, is *3* more than 9+9+2.
9+9+6=24, as 9+9+6, is *4* more than 9+9+2.
9+9+7=25, as 9+9+7, is *5* more than 9+9+2.
9+9+8=26, as 9+9+8, is *6* more than 9+9+2.

Because...... 9+8=17, is *3* less than 20,
and........... 9+8+3=20, is *3* less than 23,
then 9+8+4=21, is *3* less than 24.
9+8+5=22, is *3* less than 25.
9+8+6=23, is *3* less than 26.
9+8+7=24, is *3* less than 27.
9+8+8=25, is *3* less than 28, or *1* more than 3 eights.

Because...... 9+7+4=24—*4*=20, as 9+7 is *4* less than 20,
then.......... 9+7+5=25—*4*=21.
9+7+6=26—*4*=22.
9+7+7=27—*4*=23. Etc.

As three nines amount to 27, or three times nine, and three eights amount to three times eight or 24, etc., four eights will amount to four times eight or 32, etc. etc. Then

As............. 9+0+9=27.
then.......... 9+9+8=27—*1*=26 is *1* less than 3 nines, or 9×3.
9+9+7=27—*2*=25 is *2* less than 3 nines.
9+9+6=27—*3*=24 is *3* less than 3 nines.
9+9+5=27—*4*=23 is *4* less than 3 nines.
9+9+4=27—*5*=22 is *5* less than 3 nines.

As............. 8+8+8=24.
then.......... 8+8+7=24—*1*=23
8+8+6=24—*2*=22
8+8+9=24+*1*=25

As...... 7+7+7=21,
then... 7+7+6=21—*1*=20.
7+7+8=21+*1*=22.
7+7+9=21+*2*=23.

9+9+9+9=36.
9+9+9+7=36—*2*.
8+8+8+9=32+*1*.

6+6+6=18.
6+6+5=18—*1*.
6+6+8=18+*2*.

As 9+9+8=30—*4*=26, is *4* (*1*+*1*+*2*) less than 3 tens, then
9+9+8+8=38—*4*=34.
9+9+8+7=37—*4*=33.
9+9+8+6=36—*4*=32. Etc. Etc.

ADDITION FACILITATED BY EQUALISING NUMBERS.

DIMINISHING THE GREATER, TO INCREASE THE SMALLER,

And thus make EQUAL NUMBERS.

																	Thus
	9	8	7	6	5	4	3	9	8	7	6	5	9	8	7	9	—4=5
3 times ☞	8	7	6	5	4	3	2	7	6	5	4	3	6	5	4	5	=5
	7	6	5	4	3	2	1	5	4	3	2	1	3	2	1	1	+4=5
equal	*24*	*21*	*18*	*15*	*12*	*9*	*6*	*21*	*18*	*15*	*12*	*9*	*18*	*15*	*12*	*15*	= 15

We multiply by 3, because (3) indicates the number of figures which each column of addition contains.

	9	8	7	6	5	4	3	9	8	7	6	5	9	8	7	9
4 times ☞	8	7	6	5	4	3	2	7	6	5	4	3	6	5	4	5
	8	7	6	5	4	3	2	7	6	5	4	3	6	5	4	5
	7	6	5	4	3	2	1	5	4	3	2	1	3	2	1	1
equal	*32*	*28*	*24*	*20*	*16*	*12*	*8*	*28*	*24*	*20*	*16*	*12*	*24*	*20*	*16*	*20*

We multiply by 4, because (4) indicates the number of figures which each column of addition contains.

	9	8	7	6	5	4	3	9	8	7	6	5	9	8	7	9
	8	7	6	5	4	3	2	7	6	5	4	3	6	5	4	5
5 times ☞	8	7	6	5	4	3	2	7	6	5	4	3	6	5	4	5
	8	7	6	5	4	3	2	7	6	5	4	3	6	5	4	5
	7	6	5	4	3	2	1	5	4	3	3	1	3	2	1	1
equal	*40*	*35*	*30*	*25*	*20*	*15*	*10*	*35*	*30*	*25*	*20*	*15*	*30*	*25*	*20*	*25*

We multiply by 5, as indicating the number of figures which each column of addition contains.

	9	8	7	6	5	4	3	9	8	7	6	5	9	8	7	9
	9	8	7	6	5	4	3	9	8	7	6	5	9	8	7	9
5 times ☞	8	7	6	5	4	3	2	7	6	5	4	3	6	5	4	5
	7	6	5	4	3	2	1	5	4	3	2	1	3	2	1	1
	7	6	5	4	3	2	1	5	4	3	2	1	3	2	1	1
equal	*40*	*35*	*30*	*25*	*20*	*15*	*10*	*35*	*30*	*25*	*20*	*15*	*30*	*25*	*20*	*25*

Multiply by 5 as before, for the reason given above.

	9	8	7	6	5	4	3	9	8	7	6	5	9	8	7	9
	9	8	7	6	5	4	3	9	8	7	6	5	9	8	7	9
	9	8	7	6	5	4	3	9	8	7	6	5	9	8	7	9
7 times ☞	8	7	6	5	4	3	2	7	6	5	4	3	6	5	4	5
	7	6	5	4	3	2	1	5	4	3	2	1	3	2	1	1
	7	6	5	4	3	2	1	5	4	3	2	1	3	2	1	1
	7	6	5	4	3	2	1	5	4	3	2	1	3	2	1	1
equal	*56*	*49*	*42*	*35*	*28*	*21*	*14*	*49*	*42*	*35*	*28*	*21*	*42*	*35*	*28*	*35*

Multiply by 7, for the reason given above.

	9	8	7	6	5	4	3	9	8	7	6	5	9	8	7	9
	9	8	7	6	5	4	3	9	8	7	6	5	9	8	7	9
	8	7	6	5	4	3	2	7	6	5	4	3	6	5	4	5
7 times ☞	8	7	6	5	4	3	2	7	3	5	4	3	6	5	4	5
	8	7	6	5	4	3	2	7	6	5	4	3	6	5	4	5
	7	6	5	4	3	2	1	5	4	3	2	1	3	2	1	1
	7	6	5	4	3	2	1	5	4	3	2	1	3	2	1	1
equal	*56*	*49*	*42*	*35*	*28*	*21*	*14*	*49*	*42*	*35*	*28*	*21*	*42*	*35*	*28*	*35*

Multiply by 7, for the same reason as before.

Having seen that $N+x=T$, **and** $N=T-x=a+y$ **besides**
and also that $M-x=a$, and $M=a+x=a+x$ repeated
Now add & multiply, we get $M+N=a+T$, & $M\times N=aT+\quad xy$.

Hence. $M+N-a=T$……………………………… *A*

Therefore $N+M=(M-x)+(N+x)=a+T$. as $+x$ & $-x=0$. ……………… *B*

but $N\times M=(M-x)\times(N+x)=a\times T.+xy$,……………………………… *C*

Hence. The Sum and Product of Any Two Numbers, are based upon the addition and multiplication of the very same quantities, which are reversely equidistant from the given numbers, by ***equally diminishing and increasing the given numbers,*** or by taking from one to increase the other ; but the product of the given numbers, requires also, the product of the differences which both factors bear to either one of those equidistant numbers, as explained before. **Therefore**

The Sum and Product of Any Two Numbers, is equal to the Sum and Product of Any Two Other Numbers, which can be formed by EQUALLY DIMINISHING AND INCREASING THE GIVEN NUMBERS, **or by taking from one of them, to increase the other ; but the product requires besides, the product of the differences, which both factors bear to either one of those equidistant numbers.**

EXAMPLES WHICH ILLUSTRATE THE TERMS, COMMON TO ADDITION, MULTIPLICATION, SQUARING, &C. SHOWING THAT EQUIDISTANT NUMBERS, PRODUCED BY EQUAL ADDITION AND SUBTRACTION, FACILITATE GREAT CALCULATIONS.

As $N+x=T$ then $N=\quad T-x=a+y$
As $M-x=a$ then $M=\quad a+x=a+x$
Hence, $N+M\quad=a+T$, & $N\times M=a\times T,\quad +xy$

$83+2=85$ $\qquad 83=\;85-\cancel{2}=\cancel{6}\cancel{0}+23$
$62-2=60$ $\qquad 62=\;60+\cancel{2}=\cancel{6}\cancel{0}+\;2$
From the Sum $145\quad=145$ $\qquad 5146=5100+\quad+46$
Take 60
Leaves 85 **as the next term in addition and multiplication.**

N 4

$68-1=67$ $\qquad 68=\;67+\cancel{1}=\cancel{7}\cancel{0}-2$
$69+1=70$ $\qquad 69=\;70-\cancel{1}=\cancel{7}\cancel{0}-1$
From the Sum $137\quad=137$ $\qquad 4692=4690+\quad+2$
Take 70
Leaves 67 **as the next term in addition and multiplication.**

78−1= 77 69+1= 70 147 =147	78= 77+1=70+ 8 69= 70−1=70− 1 5382=5390 ——— 8
86−1= 85 39+1= 40 125 =125	86= 85+1=40+46 39= 40−1=40− 1 3354=3400 −46
72−2= 70 28+2= 30 100 =100	72= 70+2=30+42 28= 30−2=30− 2 2016=2100 −84
67+1= 68 46−1= 45 113 =113	67= 68−1=45+22 46= 45+1=45+ 1 3082=3060+ +22
67−2= 65 78+2= 80 145 =145	67= 65+2=80−13 78= 80−2=80− 2 5226=5200+ +26
39+30=69 29−30=-1 68 =68	39= 40−1=69−30 29= 30−1=-1+30 1131=1200 ——69
37+80=117 32−80= 2 119 =119	37= 35+2=117−80 82= 80+2= 2+80 3034=2800+ 234
687−13= 674 687+13= 700 1374 =1374	687= 674+13=700−13 687= 700−13=700−13 471969=471800+ +169

From the Sum of any two numbers or factors, taking a third one, leaves a fourth, which added to the third, produces the Sum of the two first numbers; or, multiplying the third and fourth, nearly produces, the product of the two first numbers, and is generally sufficient to satisfy a Contracted Decimal Multiplication of those numbers. But the product of the differences, which the two first numbers bear to the third, completes the multiplication, as we have already shown.

To show this logarithmic fourth number or average, see Nexsen's Short-Hand Arithmetic, page 75, for the following

METHODS OF FINDING THE AVERAGE

1	83= 60+23	68= 70−2	78= 70+8
2	$\cancel{6}2$= $\cancel{6}\cancel{0}$+ 2	69= $\cancel{7}\cancel{0}$−1	69= $\cancel{7}\cancel{0}$−1
3	145=120+25	137=140−3	147=140+7
4	60= 60	70= 70	70= 70
5	85= 60+25	67= 70−3	77= 70+7
6	85= 60+23+2	67= 70−2−1	77= 70+8-1
7	85= 83+ 2	67= 68−1	77= 78−1
8	85= 62+23	67= 69−2	77= 69+8
9	85=83+62−60	67=68+69−70	77=78+69-70
10	85=83+$\cancel{6}2$		

To find the second difference, when the first is given.

11	23=83−62+2	2=69−68+1	8=78−69−1
12	23=85−60−2	2=70−67−1	8=77−70+1

See the same Arithmetic page 82, for the following

Methods of finding the Second Difference.

37= 35+$\cancel{2}$	39= 40−$\cancel{1}$	86= 85+$\cancel{1}$	78= 80−$\cancel{2}$
82= 80+$\cancel{2}$	29= 30−$\cancel{1}$	-39=-40+$\cancel{1}$	-47=-45−$\cancel{2}$
119=115+4	68= 70−2	47= 45+2	31= 35−4
−2= −2	+1= +1	−1= −1	2= +2
117=115+2	69= 70−1	46= 45+1	33= 35−2
117=37+80	69=39+30	46=86−40	33=78−45
117=82+35	69=29+40	46=85−39	33=80−47

When $b=a$, and $y=x$, the first two formulæ, are reduced to a formula for squaring numbers. Thus, $(a+y)(a+x)=(a+x)(a+x)=(a+x)^2$, and xy becomes equal to x^2, and M equal to N ; so that, $a(N+x)+xy$ becomes $a(N+x)+x^2$, and $(M-x)(N+x)+xy$ becomes $(N-x)(N+x)+x^2$; therefore, the second formula, becomes a new formula for squaring numbers, which is

$(a+x)^2=a(N+x)+x^2$, and equal to $(N-x)(N+x)+x^2$..............Formula 10th

We are thus led at once to

CONTRACTING FORMULÆ FOR THE POWERS OF NUMBERS.

$$(a+x)^2=a^2+2ax+x^2=a(a+2x)+x^2=a((a+x)+x)+x^2=a(N+x)+x^2=(N-x)(N+x)+x^2$$

In which we see that a^2+2ax, contains the given number (N) plus x. (a) times, because, as $N=a+x$, then $N+x$ will be equal to $a+2x$; and a times $N+x$, must therefore be a times $(a+2x)$ its equal, which being multiplied, produces (a^2+2ax), the first two terms of the usual formula for squaring. And as $a+x=N$, then $a=N-x$; so that if we express $N-x$ by a, and $N+x$ by T, we will have $(a+x)^2=aT+x^2$.

By similar reasoning, we could also show that $(a-x)^2=aT+x^2$; in which case, increase the number by (x), before diminishing it by (x), which will only make a and T exchange values. Hence

The Square of Any Number, is equal to the product of any two numbers, between which it is equidistant, plus the square of that equidistance

$N-x$, and $N+x$, also show, that these equidistant numbers, a and T, are produced by *EQUALLY DIMINISHING AND INCREASING THE NUMBER TO BE SQUARED*, by x, and are therefore, *reversely equidistant* numbers.

Hence. *To Square 687.* We augment and diminish 687 by *13*, and obtain *700* and *674*, which we multiply together, and then add the square of *13*, making 471800+169=471969, the square of 687.

When x, is comparatively small, the product of *reversely* equidistant numbers, must almost equal the square required, becoming so, as the value of x diminishes. Let $x=1$, then $N^2=aT+1=a^2+2a+1=(a+1)^2$.

Notice also, that the product of *reversely* equidistant numbers, is equal to a^2+2ax, because $aT+x^2=a^2+2ax+x^2$. Hence $aT=a^2+2ax$.

We will illustrate the rules for squaring, cubing, etc., by examples hereafter.

By the last example, we show that squaring depends upon the very same principle that addition and multiplication do ; for squaring is the multiplication of equal factors, so that M and N become equal.

Hence, $M+N=a+T$, becomes $2N=a+T$, therefore $N=\frac{1}{2}(a+T)$, for

$687+687=700+674$ *equals twice* 687 ; *hence* $687=1374\div2=\frac{1}{2}(700+674)$

which shows, that the number to be squared, which is the square root of the square, is half the sum of the equidistant numbers we multiply for its square. Hence

The Square Root of a Number, is half the sum of the equidistant numbers, which multiplied, produce the Square very nearly. Therefore

Dividing any Square, say 471969. *by any number*, say *700 ; the quotient 674, and divisor 700, must be equally distant from the Root* 687.

Therefore. Half the sum of the quotient and divisor to a square, produces the Root, exactly or very nearly ; assuming this half sum for a new divisor to the given square, and taking half the sum of this new divisor and quotient, produces the root perfectly or twice as many correct figues as the divisor contains, and often one more besides, which as another new divisor, leads us to redouble the number of correct root figures ; so that, with a divisor of one significant figure, we find two or three correct root figures, which as a new divisor, produces five to seven correct root figures, and they again as another divisor evolve even to fourteen or fifteen correct root figures, and they also, evolve in the same way about thirty figures, and thus, **BY A VERY FEW DIVISIONS, FAR OUTREACH ANY KNOWN SYSTEM OF LOGARITHMS**, which we will prove with examples.

But the first divisor, must always be the nearest square root, we can ascertain to the first period, EVEN THOUGH ITS SQUARE EXCEED THAT PERIOD, and this must always be done for the first figure of any root whatever. Thus,

For the square root of 47,19,69 ; 7 squared, is nearer than 6 squared. to the period 47 ; so we divide by 7 or 700. and proceed thus,

```
                    Nearest Square Root. 700)471969  A SQUARE.
  687— 13= 674 ......................... 674    QUOTIENT.
  687+ 13=     700...................... 700    DIVISOR.
471969=169+471800. 687 SQUARED.        2)1374   DIV.+QUOT.
                                          687   SQUARE ROOT.
```

What is the square root of 785 ?........................*Answer* 28.01785+

```
          30)785    28)785      2801785)785
             26       2803571         280178529044876+
             30       28              2801785
           2)56     2)5603571       2)5603557029044876
             28       28.01785+.      28.0178514422438—Sq. Root.
Correct Square Root.............................. 28.01785144224879985—.
```

By the next similar operation, a fourth division, we would get about *thirty correct root figures*, and a fifth division would yield at least *sixty* of them, and thus, ***FAR OUTREACH THE LIMITS OF LOGARITHMS*** ; for, by one very small division, we find the square root of 471969 to be 687, sooner than by logarithms, or by any other system of square roots ; and by two very small divisions, we find the root of 785, to be 28.01785, which is about as far as any table of logarithms venture ; by a third division we go beyond the limits of logarithms, and farther and farther in those which follow, doubling and redoubling extra root figures by each succeeding division. Whereas, by the usual methods, only one figure at a time can be found ; but this method evolves a great many at a time, hence its usefulness is greater than that of any other.

Notice. Extra root figures can be often found, by taking half of the difference between the quotient and divisor, as in the last example.

If we diminish the given square, by the square of an assumed root, and divide the remainder, by twice that assumed root, we will get extra root figures, prevent repetitions, and make shorter divisions.

Remember also, that all these divisions can be vastly curtailed by Contracted Division.

We have seen on page 23, that the square root of any number, is half the sum of the equidistant numbers, which multiplied together, produces the square, or nearly produces it, whether a little too great or a little too small.

So, the cube root of any number, is one third of the sum of any three factors which are proportionally equidistant from the root; whether two of them are equal or not; providing that the product of all three of the factors, equals or nearly equals the cube.

Because the average of any three numbers, is a third of their sum, and

The average of any three factors, is one third of their sum also.

And the average of any number of numbers, or number of factors (equal or unequal) is their sum, divided by the number of those numbers, or number of factors, and thus,

By averaging the assumed factors of any power, we can find any required root. For the root of any power, is the average of several other roots which may be assumed. Thus

$a^{11} \div a^6 = a^5, \div a^3 = a^2$. Therefore $a^{11} = a^6 + a^3 + a^2$. Hence $a = (6a + 3a + 2a) \div 11$

$$\begin{array}{l} a^6)\underline{a^{11}} \\ a^3)\underline{a^5} \end{array}$$

THE QUOTIENT a^2 *contains 2 equal factors, or* $2a$.
THE DIVISOR a^3 *contains 3 equal factors, or* $3a$.
THE DIVISOR a^6 *contains 6 equal factors, or* $6a$.
ADDING. a^{11} *contains 11 equal factors, or* $11a$.
THEREFORE a EQUALS $11a \div 11$, EQUALS $(6a + 3a + 2a) \div 11$.

As the product of any two or more numbers contains their factors, the product will also contain the sum of those factors, thus 49, which is the product of 7 times 7, contains also 14, the sum of the factors 7 and 7. Because the greater contains the lesser.

So any power will contain as many equal factors, as its exponent indicates, and will therefore contain the product of its root and exponent. Thus a^3 contains $3a$, as it contains its equal $a + a + a$

But when two or more powers are multiplied together, the index of their product will be the sum of the exponents of the powers just multiplied, but each power contains the product of its root and exponent, therefore

The product of two or more powers, *contains the sum of the products which the powers contain, composed of their roots and corresponding exponents multiplied together,* and thus

We notice, that while the exponents are added for the index of the product of two or more powers, *these exponents of the powers are multiplied by their roots, because the powers contain their products.*

So that while adding the exponents, for the index of the power of a product; each of those exponents is multiplied by the root it indicates, for the product which that power contains.

We therefore by adding the products of the roots and corresponding exponents, necessarily involve the addition of the exponents answering to the power of the product of those powers.

And as a power contains the product of its exponent and root, the addition of the exponents, necessitates at the same time, the multiplication of those exponents by the roots of the powers, to satisfy the product which the powers contain; for, if in a series of multiplications, some of the factors be added, the products must necessarily be added.

TO FIND THE ROOT OF ANY POWER.

Point off the given power, into periods of as many figures as the index of the required root indicates, beginning at units, and counting off to the left. But when there are decimals, count off both ways, from the decimal point. The highest period in a whole number may sometimes contain only one or two figures.

*By trial, find the nearest root figure or number, to the highest period of the given power, **even though that period is exceeded.*** Not taking the nearest root IN the period, (which has always been the way heretofore), but always taking the nearest root TO that period, which we find by comparing the similar powers of the supposed roots, TO that period; and whichever is ***the nearest in approximation, whether greater or less,*** take its root in preference to any other. Doing so, enables us to find double and trible the usual number of root figures at once.

Assume for a divisor, this figure or number raised to a power designated by one half of the index of the required root; (when that index is an odd number like 11, take its greatest half 6, as the exponent),

DIVIDE, *and consider the quotient, as of that power, which the other half of the index indicates, and extract its root according to that exponent, for about twice as many figures as have already been found, and make a new divisor to this quotient, as before, by raising the new root to a power designated by one half of its index.*

DIVIDE, *and find the root of this quotient as before; repeat these operations, until the sum of the exponents of the powers of the divisors and **last quotient,** equal the index of the required root.*

*Multiply the root of each divisor and of the last quotient, by the index of its power, **and divide the sum of these products,** by the index of the required root, and this **quotient will be the root required, or a very near approximate to it.***

For further decimal root figures, involve the supposed root half way, as before, for a more perfect divisor to the given power, and repeat the operations of the rule.

Lengthy divisions are unnecessary, dividing only far enough to get two or three figures of the supposed root, then by the next division getting twice as many, and so on, doubling the number of root figures each time, or one more than the double. Thus

$a^{11} \div a^6 = a^5,\ \div\ a^3 = a^2$. Therefore $a^{11} = a^6 + a^3 + a^2$. Hence $a = (6a+3a+2a) \div 11$

$a^6)a^{11}$

$a^3)a^5$

THE QUOTIENT a^2 *contains 2 equal factors, or 2a.*
THE DIVISOR a^3 *contains 3 equal factors, or 3a.*
THE DIVISOR a^6 *contains 6 equal factors, or 6a.*
ADDING. a^{11} *contains 11 equal factors, or* 11a.
THEREFORE a **EQUALS** *11a*÷11, **EQUALS** (6a+3a+2a)÷*11.*

What is the eleventh root of 11 ? *Answer* 1.2436, which is a trifle too great.

It is evident that the root stands between 1 and 2, or 10 and 20 ; we therefore try 11 or 12 as the root, and make its sixth power a divisor.

$(11)^6=1771561)11.$
$(14)^3=\ 2744)6,209$ its 5th root is 1.4
$(146)^2=\ 2.1316=\ 2.15,$ its 2nd root is 1.46
146 × *2 = 292*
14 × *3 = 42*
11 × *6 = 66*
11)1372
1.24 **THE ROOT.**

$(12)^6=2985984)11.$
$(13)^3=\ 2197)\ 3.6838777$ its 5th R.=1.3
$(129)^2=1.6641=\ 1.676776$ its 2nd R.=1.29
129 × *2 = 258*
13 × *3 = 39*
12 × *6 = 72*
11)1368
1.2436= **ROOT CONTINUED.**

What is the seventh root of 12311,7154813,2409344..............Answer 384.

$(4)^7$=16384 is very much nearer to the period 12311, *than $(3)^7$=2187 ; we therefore assume 4 for the nearest root, even though its seventh power exceeds the period* 12311. *We always take the NEAREST TO a period, EVEN THOUGH IT EXCEED that period.*

We therefore divide by $(4)^4$,=256)12311,7154813,2409344
Cube root of...............................48092639 =363,× *3=1089*
4 × *4=16*
7) 2689=384. *Answer.*

What is the fifth root of 36936242722357 ? ....*Answer* 517.
3693,62427,22357÷*125 for* $(5)^3$; =295490, *its square root is 543,×2=1086*
500 ×*2=* [illegible]
Answer 517

What is the fifth root of 1256? *Answer* 4.16676.

1256÷64 *for* $(4)^3$, =19.625, *its nearest sq. root is* *443*, ×2= 886
The assumed root is *400*, ×3= 12
Still nearer root by averaging, thus 416, =5) 2086

Cube 4.16 *for a new divisor* 71.991296.
1256 ÷ 71.991296 = *17.4465535*, *its sq. root is* *41769*, × 2 = 83538
416 × 3 = 1248
5) 208338=4.16676 *Ans.*

What is the cube root of 0.250?........................ *Answer* 0.62996052+

.250÷36 *for* $(6)^2$ = 69
60×2=120
189÷3=63

.250÷$(63)^2$ *or* 3969=62988159
63×2= 126
18898815÷3=0.6299605+

What is the fourth root of 1040,8624,5376........................*Answer* 568.

6 is the nearest root to 1040, *then*

104086245376÷$(600)^2$=10408624÷36=289128,=$(537)^2$ *nearly*, ×2=1074
600×............2=1200
2274÷4=568.

By formulæ A B and C on page 19, and examples on that and the following page, we see plainly that the product of any two numbers not only contains their factors, but also necessarily *contains the Sum of those factors;* and this is equally true of the product of several numbers, or of the product of several powers.

In squaring 687 on page 22, we find by the same example on page 20, that *1374*, the sum of the equidistant factors *674* and *700*, is equal to twice 687, the number to be squared.

So that doubling any number to be squared, we find the sum of the factors, while their product equals or nearly equals the required square. In the same manner, tribling any number which is to be cubed, we find the sum of the factors, while their product equals or nearly equals the required cube.

Therefore, *Multiplying any number, by the index of its power, shows the sum of the factors of that power.*

And this is the reason why the sum of the factors of a power, is divided by the index of that power, to find the root, which is the average of those factors.

But, by formula *(A)* M+N—*a*=T on page 19, and the last example on page 20, and conclusion drawn therefrom on page 21; diminishing the sum of the factors of a product, or of a power (which is always a product of factors) by any of its factors, or by the sum of several of its factors, the remainder will contain the sum of the other factors which remain, and we therefore divide that quantity by the number of the factors which it contains, to find the other factor of that power, all of which together should average the number to be involved. Hence

TO RAISE ANY NUMBER TO ANY POWER.

Assume any number, which can be more easily raised to the required power, (even though it be greater than the given number).

Note the difference between the given and assumed numbers.

Multiply the given number, by the index of the required power.

Multiply the assumed number by half the index of the required power (or its greatest half, when that half is an odd number).

Divide the difference of the products just found, by the difference ~~of~~ of the multipliers of the given and assumed numbers.

Raise the quotient thus found, to the power indicated by the divisor which produced it.

Raise the assumed number to the power by which it was multiplied.

Multiply these two powers together. and allow for the difference between the given and assumed numbers, as in squaring, cubing, etc. See the author's Treatise on the Powers Of Numbers.

FOR VERY HIGH POWERS.

For very high powers, assume several numbers which shall be easier to raise to powers with indices found by taking half of the index of the required power, and then one half of what remains and one half again of what further remains, until the last half becomes very small and easy to involve. Always taking the greatest half when the index of a power is an odd number.

Multiply the assumed numbers, by their indices, and subtract the sum of their products, from the product of the given number multiplied by the index of the required power, and divide the remainder by the index of the powers which remain.

Raise this quotient to a power indicated, by the divisor ; and raise the assumed numbers to powers which their multipliers indicate, then multiply the powers just found, and allow for the difference, in the way explained by the author's Treatise on The Powers and Roots of Numbers.

This treatise introduces new formulæ, which will be far more useful to the mathematician than the Newtonian, containing but half as many terms as they require, with far greater power and brevity ; and this is the reason that many root figures are found all together at once in each operation, as the examples already given have shown, instead of finding only one figure at a time by the usual process.

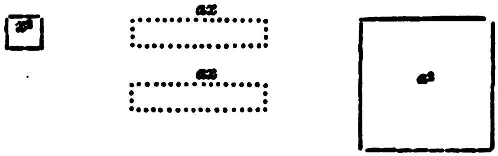
The laws for finding the coefficient and exponent of any term, with the proper sign, are clearly explained and are very interesting.

N. B. It will very frequently be found, that the difference to be allowed for, can be disregarded, especially when the left hand figures only are required, to obtain an approximation which will satisfy the answer, and thus avoid enormous calculations which are useless ; or when the verification of the supposed root of any number is desired.

GEOMETRICAL ILLUSTRATION OF THE PRINCIPLE OF EQUAL DIMINUTION AND INCREASE OF QUANTITIES.

DEMONSTRATING ITS USE IN SQUARING.

Let us take two square blocks marked a^2 and x^2, and two rectangular blocks marked ax with sides equal to one side of a^2 and one side of x^2.

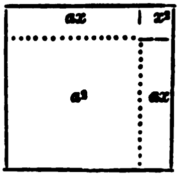


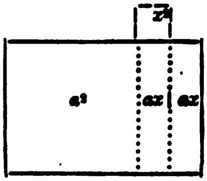
By different combinations of the above four blocks, we form the two following figures, which must be equal in area, because they are both composed of the very same blocks; and notice particularly that by simply taking the block ax from the top of the first figure and adding it to the side of it, we form the second figure.

So that by the principle of an equal amount of subtraction and addition, that is by adding to one side what is taken from the other, we still maintain an equal area in the surface of the two figures.

Fig. 1. *Fig. 2.*

N^2 *EQUAL TO* $aT+x^2$





Let $N=$ $a + x$. Let $T=$ $a + x + x$ $=a+2x$

$N=$ ———————— and $T=$ ———————— $=a+2x$

Then $a =$ ———————— and $T=$ ———————— are the sides of the rectangle aT; but aT *equals* $a(a+2x)=a^2+2ax$; *adding* x^2 we find that $aT+x^2 = a^2+2ax+x^2= (a+x)^2 = N^2$

T *Longest side of the Rectangle* $\surd$————————

N *Side of the Square.* MEAN PROPOR. ————————

a *Shortest side of the Rectangle.* ————————

Notice. that the smaller, x^2 becomes, the more the second figure becomes a square, because the blocks marked *ax* narrow down in width according as x diminishes ; this is the reason why

The Area of any Square, *is equal to the Area of any Rectangle whose longest side is as much longer as its shorter side is shorter, than one side of the given square ; PLUS the Area of the Square whose sides equal the difference between one side of the given square and either side of the rectangle* (aT).

Hence, the smaller this difference, the smaller the square becomes which is based upon it, and the more the area of such rectangles equal the area of the given square, and may therefore be assumed to be equal for many purposes.

Therefore on page 22, we conclude that *The Square of Any Number, is equal to the product of any two numbers between which it is equidistant, plus the square of the equidistance.*

Again, **The Area of Any given Square,** *is equal to the Sum of the areas of any Rectangle and Other Square, when half the sum of the sides of the rectangle, is equal to one side of the given square ; when also, the difference between one side of the given square and either side of the rectangle, forms the sides of the additional square.*

Therefore, *To find the Square Root of a Number, which is one side of a square ; take half the sum of the divisor and quotient, because they correspond to the longest and shortest sides of the rectangle ;* see page 23 ; and because

Geometrically, *the side of the given square is a* MEAN PROPORTIONAL *between the longest and shortest sides of the rectangle* (aT), as the lines marked T, N, and *a*, indicate, at the foot of page 29.

The difference between the squares of any two numbers, *is equal to the square of their sum, minus twice the product of their sum, multiplied by the smallest of those given numbers ; or is equal to the square of their difference, plus twice the product of their difference, multiplied by the smallest of those given numbers.* Thus

$$\begin{array}{lcl}
a^2+2ax+x^2 & = & (a+x)^2 \\
\quad -2ax & = & -2ax \\
\hline
a^2 \qquad +x^2 & = & (a+x)^2-2ax \\
\qquad -2x^2 & = & -2x^2 \\
\hline
a^2 \qquad -x^2 & = & (a+x)^2-2ax-2x^2 \\
 & = & (a+x)^2-2x(a+x)
\end{array}
\qquad\qquad
\begin{array}{lcl}
a^2-2ax+x^2 & = & (a-x)^2 \\
\quad +2ax & = & +2ax \\
\hline
a^2 \qquad +x^2 & = & (a-x)^2+2ax \\
\qquad -2x^2 & = & -2x^2 \\
\hline
a^2 \qquad -x^2 & = & (a-x)^2+2ax-2x^2 \\
 & = & (a-x)^2+2x(a-x)
\end{array}$$

To Contract the Formula for Cubing Numbers. We reduce $(a+x)^3=a^3+(3a^2x+3ax^2)+x^3$, to $a^3+3ax(a+x)+x^3$, then to $a^3+3axN+x^3$. 11th. $(a-x)^3$, will, by like reasoning become $a^3-3axN-x^3=a^3-(3axN+x^3)$.

Hence. *The Cube of Any Number, is equal to the cube of any other number,* (a), *which can be more readily cubed ; plus or minus, the amount obtained by adding the cube of their difference* (x), *to three times the continued multiplication of the given number, the assumed number, and their difference.* **Subtracting that amount only when assuming too great a number for convenience sake.**

To Cube 687; which we notice is 13 *less than* 700.

From the difference between the cubes of 700 and of 13, take *three* times the product of 700×687×13; thus

687 = 700 — 13,

$(700)^3—(13)^3$ = 343000000—2197 = 342997803

Tripling 687 × 700 × 13, = 187551.. to subtract.

687 cubed equals....................324242703

Or, adding the cube of 13, to three times 700×687×13; take their amount 18757297, from the cube of 700, leaves the cube of 687.

To Cube 713; which we notice is 13 *more than* 700.

Add the cubes of 700 and of 13, to three times 700×713×13; then 343002197 + 19464900 = 362467097 equals 713 cubed.

The difference between the cubes of any two numbers, *is equal to the cube of their difference, plus three times the product of those numbers, multiplied by their difference.*

$a^3—3axN—x^3=N^3$. Therefore $a^3—x^3=N^3+3axN$.

Let a^3= $(400)^3$ = 64000000

Let x^3= $(383)^3$ = 56181887

$N^3=(17)^3=4913$. + 3 times 400×383×17 = 7818113

N^3 + 3 times a× x× N

The sum of the cubes of any two numbers, *is equal to the cube of their sum, minus three times the product of those numbers, multiplied by that sum.*

$a^3+3axN—x^3=N^3$. Therefore $a^3+x^3=N^3—3axN$.

Reducing the Formulæ of Higher Powers.

$(a+x)^4=a^4+(4a^3x+6a^2x^2+4ax^3)+x^4=a^4+4ax(a^2+1\frac{1}{2}ax+x^2)+x^4$; in which we see. that $a^2+1\frac{1}{2}ax+x^2=a^2+2ax+x^2-\frac{1}{2}ax=(a+x)^2-\frac{1}{2}ax=N^2-\frac{1}{2}ax$; and substituting this value, we find that

$(a+x)^4=a^4+4ax(a^2+1\frac{1}{2}ax+x^2)+x^4=a^4+4ax(N^2-\frac{1}{2}ax)+x^4$. .... 12th.

$(a-x)^4$, will likewise become $a^4-4ax(N^2+\frac{1}{2}ax)+x^4$.

$(a+x)^5=a^5+5a^4x+10a^3x^2+10a^2x^3+5ax^4+x^5$; by re-arranging the terms, we get

$a^5+(5a^4x+5ax^4)+(10a^3x^2+10a^2x^3)+x^5$, which we reduce to

$a^5+5ax(a^3+x^3)+10a^2x^2(a+x)+x^5$. .... A.

But $a^3+x^3=(a^3+3axN+x^3)-3axN=(a+x)^3-3axN=N^3-3axN$; and substituting this value, and N for the value of $a+x$, we reduce the long expression $5ax(a^3+x^3)+10a^2x^2(a+x)$ found in equation A to $5ax(N^3-3axN)+10a^2x^2N$, which is equal to $5ax(N^3-3axN+2axN)$; reduced again, becomes $5ax(N^3-axN)=5axN(N^2-ax)$; Hence

$(a+x)^5$ becomes equal to $a^5+5axN(N^2-ax)+x^5$. .... 13th.

$(a-x)^5$ will become equal to $a^5-5axN(N^2+ax)-x^5$, by like reasoning.

By multiplying the contracted formulæ of the second and third powers, we would produce that of the fifth just evolved ; and in the same way, multiplying the contracted formulæ of third and fourth powers produce that of the seventh ; the square of the second produces the fourth ; the square of the cube producing the sixth, etc. And thus we shall find any power of any number ; and in order to show the law of their development and harmony, let us remove the parenthesis, so as to make this very evident.

We can then see in the table on the next page, that

The coefficient of the third term of any Power, is equal to the sum of the coefficients of the second and third terms of the next lower Power, minus unity.

Thus, in the sixth power, the sum of the coefficients 6 & 9 equal 15, minus a unit equal 14, which we see is the coefficient of the third term of the seventh power.

Many other curious features can be shown, but we will pass to general laws.

RECAPITULATION OF CONTRACTED FORMULÆ, FOR FINDING ANY POWER.

When $N=a+x$, then

$$\begin{array}{lcl}
N^2=a^2+2ax,\ldots\ldots\ldots\ldots+\ldots\ldots\ldots x^2 & = & a^2+2ax\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots+x^2 \\
N^3=a^3+3ax\times N,\ldots\ldots\ldots+\ldots\ldots\ldots x^3 & = & a^3+3axN\ldots\ldots\ldots\ldots\ldots\ldots\ldots+x^3 \\
N^4=a^4+4ax\times(N^2-\tfrac{1}{2}ax)+ \quad x^4 & = & a^4+4axN^2-2a^2x^2 \quad +x^4 \\
N^5=a^5+5axN(N^2-\ ax)+ \quad x^5 & = & a^5+5axN^3-5a^2x^2N \quad +x^5 \\
N^6=a^6+6axN^2(N^2-1\tfrac{1}{2}ax)+2a^3x^3+ \ x^6 & = & a^6+6axN^4-9a^2x^2N^2+2a^3x^3 \ +x^6 \\
N^7=a^7+7axN^2(N^2-\ 2ax)+7a^3x^3N+x^7 & = & a^7+7axN^5-14a^2x^2N^3+7a^3x^3N+x^7
\end{array}$$

Let us first examine the law governing the formation of coefficients, and then the laws for the exponents, and the signs.

THE LAW FOR FINDING COEFFICIENTS

The first and last terms. do not require coefficients as the figure (1), can always be understood without specifying it.

Form as many columns of numbers, as half the index of the required power calls for. When that index is odd, take its smallest half.

Now make the simple numbers 2, 3, 4. etc., form the first column, the last number being the index of the required power.

Then add the column downward, in this way ; to the sum of the two first numbers, add the third ; and to the sum of these three, add the fourth ; and to the sum of these four, add the fifth ; proceeding in this manner, to add to each summation the next number only, and continuing so, until all but the two last numbers are added ; recording at the same time, the additions as they are made, in a second column to the right of the first, beginning it however with the figure 2.

The second column will then contain two numbers less than the first.

Add the second column in the same manner as the first, to form a third ; and this likewise, to form a fourth, etc.

Begin every column with the number 2 ; and make each column, contain two numbers less than the one from which it is derived.

When a column contains only three numbers, the next one should contain nothing else but the number 2.

The last numbers in these columns, will be the co-efficients of all the terms which are between the first and last.

Thus the columns at the top of the next page, will illustrate the co-efficients of the 9th, 8th, and 6th powers.

For the 9th, we see 9, 27, 30, and 9.

For the 8th, we see 8, 20, 16, and 2.

For the 6th, we see 6, 9, and 2.

So that we make

$$(a+x)^9 = a^9+9axN^7-27a^2x^2N^5+30a^3x^3N^3+9a^4x^4N+x^9.$$

$$(a+x)^8 = a^8+8axN^6-20a^2x^2N^4+16a^3x^3N^2+2a^4x^4+x^8.$$

2	2	2	2	2	2	2	2	2	2	2
3	5	7	9	3	5	7		3	5	
4	9	16		4	9	16		4	9	
5	14	30		5	14			5		
6	20			6	20			6		
7	27			7						
8				8						
9										

Remember, that the figure 2 heads up every column, and preceeds the record of additions to follow it.

Notice also, that the coefficient of the next to the last term of an even power is always 2, while that of the odd powers, is always the index of the required power, and same as that of the second term.

We formed the second column, thus,

2+ 0= 2 to head up the new column.
3+ 2= 5 equal to the sum of 2+3,.................. of the first column.
4+ 5= 9 equal to the sum of 2+3+4,............... of the same column.
5+ 9=14 equal to the sum of 2+3+4+5............of the same.
6+14=20 equal to the sum of 2+3+4+5+6.
7+20=27 equals.................2+3+4+5+6+7.

We form the third column from the second, thus,

2+ 0= 2 to head the next column.
5+ 2= 7 equals 2+5 the sum of the first two numbers.
9+ 7=16 equals 2+5+9 the sum of the first three numbers.
14+16=30 equals 2+5+9+14 the sum of the first four numbers.

TABLE, SHOWING THE COEFFICIENT OF ANY TERM, TO BE The Summation Of An Arithmetical Series.

Exponents of N.	*Coefficients of 1st Terms.*	*Coefficients of 2nd Terms.*	*Coefficients of 3rd Terms.*	*Coefficients of 4th Terms.*	*Coefficients of 5th Terms.*	*Coefficients of 6th Terms.*	*Coefficients of 7th Terms.*	*Coefficients of 8th Terms.*
		ax	a^2x^2	a^3x^3	a^4x^4	a^5x^5	a^6x^6	a^7x^7
N	1	2	2	2	2	2	2	2
N^1	1	3	5	7	9	11	13	15
N^2	1	4	9	16	25	36	49	64
N^3	1	5	14	30	55	91		
N^4	1	6	20	50	105	196		
N^5	1	7	27	77	182	378		
N^6	1	8	35	112	294	672		
N^7	1	9	44	156	450			
N^8	1	10	54	210	660			
N^9	1	11	65	275	935			

This table is produced by raising the columns which contain the third, fourth, and other terms of the powers, so that they shall stand up even with the column of the second terms, and their first figure 2 range along on the same line.

We place the letters of the terms, to the top and side of the table, in order to see better the coefficients. **And now observe, that**

The Sum of Any Number of Numbers, from the top of Any Column, can be found in the next column, at the right of the last of the numbers added, and on the same line. Thus, 30, the coefficient of the fourth term of the ninth power, stands in the column of fourth terms, and is the sum of the series 2, 5, 9, & 14, which stand in the column of third terms ; and which, if continued for two more numbers, produces 27 as the coefficient of the third term of the same power ; and which is also the sum of another series 2, 3, 4, 5, 6, & 7, contained in the column of second terms ; which being continued for two more numbers shows 9, to be the coefficient of the first term of the same power. Therefore,

The Coefficient of Any Term, of Any Power, is the sum of an arithmetical series of summations, which continued two numbers further, produces the coefficient of the preceeding term, which is also the sum of another series still preceeding it.

Therefore, by the addition of the series 2, 3, 4, 5, 6, 7, etc., to find a coefficient, we evolve a series of summations, and the sum of all but the two last of these, produce the next coefficient ; and while the sum of any series, or the sum of any series of summations is saught for, the addition necessary, evolves a new series, and its addition, still another, and so on : but by a decrease of two summations to each series as evolved, the number of these series formed, and the whole number of coefficients possible is attained.

2+ 0= 2+ 0= 2+0=2
3+ 2= 5+ 2= 7+2=9
4+ 5= 9+ 7=16
5+ 9=14+16=3
6+14=20
7+20=27
8
9

Thus the table is formed, and so are the coefficients 9, 27, 30, & 9, of the ninth power.

2+0=2
3+2=5
4
5

5 & 5, would be the coefficients of the fifth power. Etc.

To find the coefficient of any term of any power from the table.

Look for the column containing the required term, and take the sum of the exponents of ax, a^2x^2, or a^3x^3, (which head that column), from the index of the required power, to show the exponent of N, and on the line of N with that exponent, and in the column of the given term, will be found the coefficient required. Thus,

For the coefficient of the third term of the seventh power, we find a^2x^2, and taking 4, the sum of their exponents from 7, leaves 3, for the exponent of N, then on the line marked N^3, and in the column of the third term, we find 14. Then $14a^2x^2N^3$ is the complete third term.

LAW GOVERNING EXPONENTS.

In the first place, the sum of the exponents of any term, is equal to the index of the power required.

The exponents of (a) and (x), in the first and last terms, will always be the same as the index of the power required.

The product of (ax) will appear next, in the second terms, common to all the powers, and therefore a constant quantity.

a^2x^2 the square of ax, will appear in the third term, common to all the powers.

a^3x^3 the cube of the product (ax), follows next as a constant, etc.; proceeding thus, until the product (ax) is raised to a power indicated by half of the index of the power required, or to its least half, when that index is odd, as $9a^4x^4N$ contains the fourth power of ax, and this will be its highest power.

The exponent of N, first apyears in the cubes, as a factor to ax.

The exponent of N, in the second terms, will always be two less than the index of the power required. We may designate the power required as (p), then the first exponent of N, will appear in the second terms as N raised to ($p-2$) power. The exponent of N in each succeeding term, will diminish by 2; so that, as $p-2$ represents the highest and first exponent of N, ($p-4$) will represent the second exponent of N, to be found in the third term; and ($p-6$) the next lower to be found in the fourth term, etc., until the powers of N vanish.

Notice.—while (ax) is common to the second terms of all the powers, the exponents of N increases steadily, beginning at the cube, thus we have $3axN$, $4axN^2$, $5axN^3$, $6axN^4$, etc.

While (a^2x^2) is common to the third terms of all the powers, the exponents of N increase, beginning at the fifth power, etc.; and so on; the exponents of N advance by unity, while ax, a^2x^2, a^3x^3, etc., remain constant.

Notice, that in all the powers of ($a+x$), the signs of their third terms are minus, while those of all the other terms are plus.

But in the powers of ($a-x$), the last terms have minus for the odd, and plus for the even powers. But the sign of the second terms will be minus, and the third plus, and all the following terms which preceed the last, will have minus signs.

So that the signs in the powers of ($a-x$), will be the reverse of those for the powers of ($a+x$), with the differences noted above.

Notice in these new formulæ, we only require half as many terms and coefficients as those of Newton's; and therefore gain power by the condensation of terms, which is lost by their expansion.

There are 13 coefficients in Newton's formula of the 12th power viz; 1. 12. 66. 220. 495. 792, 924. 792. 495. 220. 66. 12. 1; and only 7 small numbers for the new, viz; 1. 12. 54. 112. 105. 36. 2.

Having treated formula No 2 on page 11, so fully; the reader will probably discover how to apply the others.

By making xy the first and leading term in all these formulæ, another principle and mode of reckoning is deduced, which is amply treated upon, in *Nexsen's Short-Hand Arithmetic.*

To Superintendent.

In order to lighten the labor of teaching Arithmetic, and make it a much pleasanter study than it now is ; two principles are necessary, each of which greatly facilitate many operations.

By the first of these, we facilitate Addition, Multiplication, the Powers and Roots of Numbers, Compound Interest and Discount, Insurance, Annuities, Geometrical Progression, Mensuration, and many other branches of Arithmetic, with all the Divisions and Operations which are the reverse of any of these rules.

By the second principle, we facilitate Simple and Compound Subtraction, Division, certain kinds of Multiplication, and many other branches.

The Advantages of this System, are

First. The minds of the scholars are not distracted by numerous rules, ideas, methods etc.

Second. Different classes in school, can learn different branches of Arithmetic, by the same principle.

So that, when a scholar in a class on Addition, is promoted into a class on Multiplication, he learns to perform multiplication by the same principle by which he was taught to add numbers.

And, as the scholar from a class on Multiplication, passes up into classes upon the higher branches of Arithmetic, he still finds that the same principle enables him to solve the higher operations of Squaring, Cubing, Square Root, Cube Root, Insurance, etc.

So that, **DIFFERENT CLASSES, CAN LEARN DIFFERENT BRANCHES OF ARITHMETIC, BY THE SAME PRINCIPLE.**

For, by taking from one number to increase another, we can produce easier numbers to add or multiply, and thus simplify Addition, Multiplication, Involution, etc.

Third. Though so much can be done by each principle ; they are so simple, that even the primary scholar can understand them.

The primary steps to learn are, first, that 9 & 1, 8 & 2, 7 & 3, 6 & 4, and 5 & 5 make 10 : and then that 1, 2, 3, etc., taken from 9, 8, 7, etc., leave —,

Now begin to teach the principle in its application to figures first, and then to numbers, thus

To add two figures like 6 & 8. Take from one figure to increase another to 10, produces easier numbers to add ; thus, take 2 from 6, and add it to 8, make **4 and 10, fourteen** ; they can see at a glance that 10 and 4 make 14, *by the similarity which the new numbers* 10 *and* 4, *bear to their sum* 14 ; *and learn also by the sound that* 4 *and* 10 *are fourteen.*

They can next learn to add *several figures*, then *two numbers*, and finally *several numbers*, in the same way. Thus, it is much easier to add 10+10+10+3=33, than it is to add 9+9+8+7=33, by simply taking from 7, to make the other figures, 3 tens. Then 30+3=33.

$$\begin{array}{r} 9+9+8+7=33. \\ 1+1+2-4=0 \\ \hline 10+10+10+3=33. \end{array}$$

Then to teach Multiplication by the same principle, let the scholar see that we produce easier numbers to multiply, by simply taking from one factor to increase the other ; and he will thereby recognize the same principle, and a similarity of terms in Addition and Multiplication.

And to Square numbers, tell him to do the very same thing which he did in multiplication.

To teach him the Powers and Roots of Numbers, show him that the terms by which he performed Addition, Multiplication, and Squaring, could also be found, by simply taking one of these terms from the sum of the given numbers or factors, leaving the other term for addition or multiplication, as the case may require.

So that the sum of the new numbers or new factors, will always equal the sum of the original numbers or factors.

Therefore, to find the Square Root of a number : we divide the sum of the factors (divisor and quotient), which a square contains, by 2, because it should contain two equal factors.

In the same way, we divide the sum of the three factors which a cube contains by 3, for the cube root, etc.

To teach Mensuration, Compound Interest and Discount, Geometrical Progression, etc. ; apply this principle, and this new method of Powers and Roots of Numbers.

The second Principle is simply this, that to diminish two numbers equally, or to increase them equally, does not alter their difference, and will enable us to produce easier numbers to subtract, both in Simple and Compound Subtraction, and also produce easier numbers to multiply, etc.

These principles can be taught by simple illustrations, to make them interesting.

The author would express his thanks to many kind friends, and especially to Messrs J. H. ZABRISKIE Principal of Grammar School No 16, ALONZO HOPPER Principal of Grammar School No 21, B. D. L. SOUTHERLAND Principal of Grammar School No 3, *(all of N. Y. City)* for suggestions as to the mode of introducing these principles into the schools ; and to G. H. WHITE, Principal of Grammar School No 70, for the earnest arguments which he urged upon members of the School Book Committee of the Board of Education of N. Y. City, for the adoption of NEXSEN'S Short-Hand Arithmetic, as a text book.

The following text books, explain these new methods and principles.

Nexsen's Short-Hand Arithmetic, is designed especially for all grades in the Grammar Schools.

Nexsen's Algebra & Arithmetic, is designed for Teachers, for the higher classes of Grammar Schools, and especially for Colleges, and Academies.

These works are not intended to superceede others now in use, but are important auxilliaries which extensively simplify the instruction of these branches.

Any assistance that can be rendered towards the introduction of this system in the Schools would be gladly offered.

Yours Truly

H. M. Nexsen.

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